

Coherent Pion Production by neutrinos and some implications.

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Precise Neutrino-Hadron Interactions

Among the topics to be covered today will be

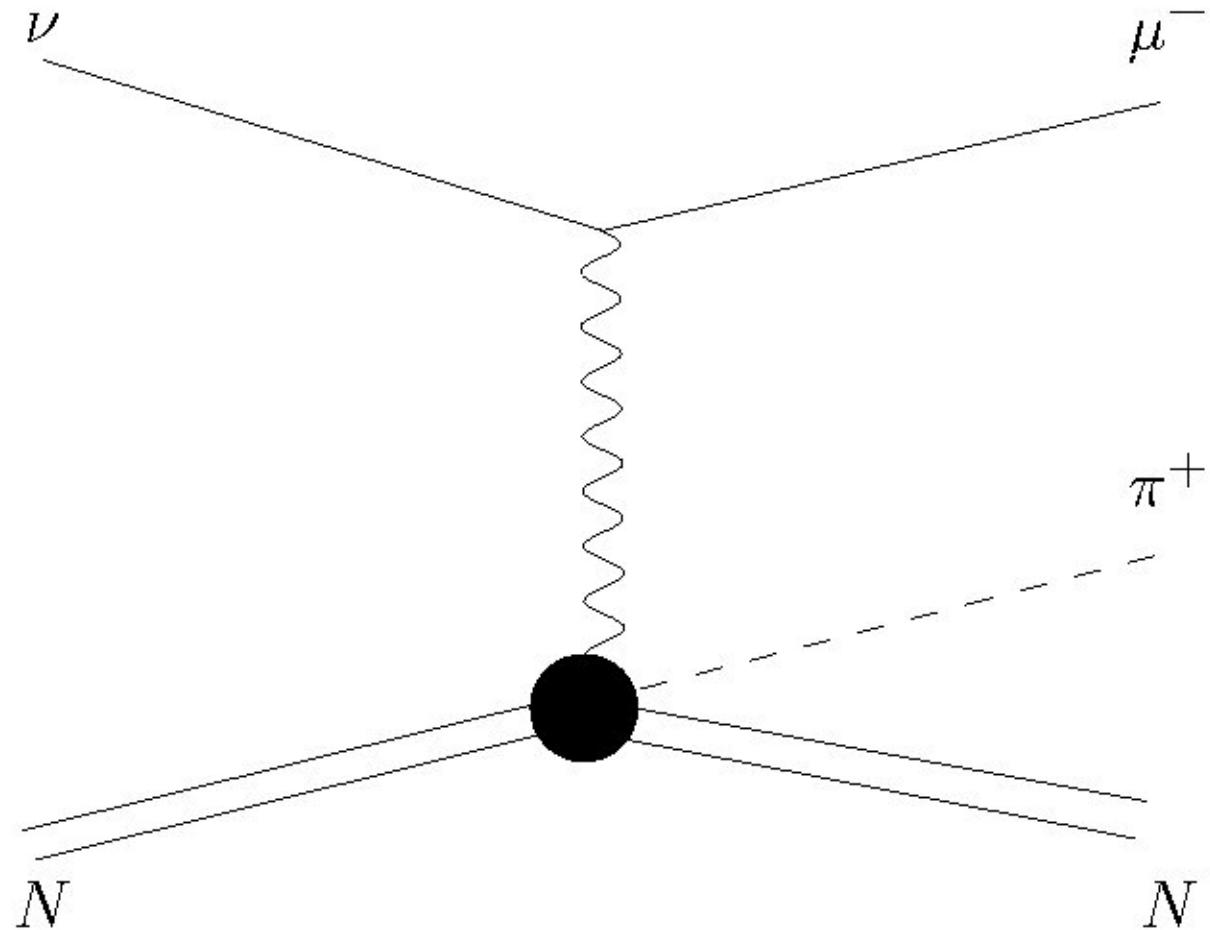
A historical reminder how this topic developed.

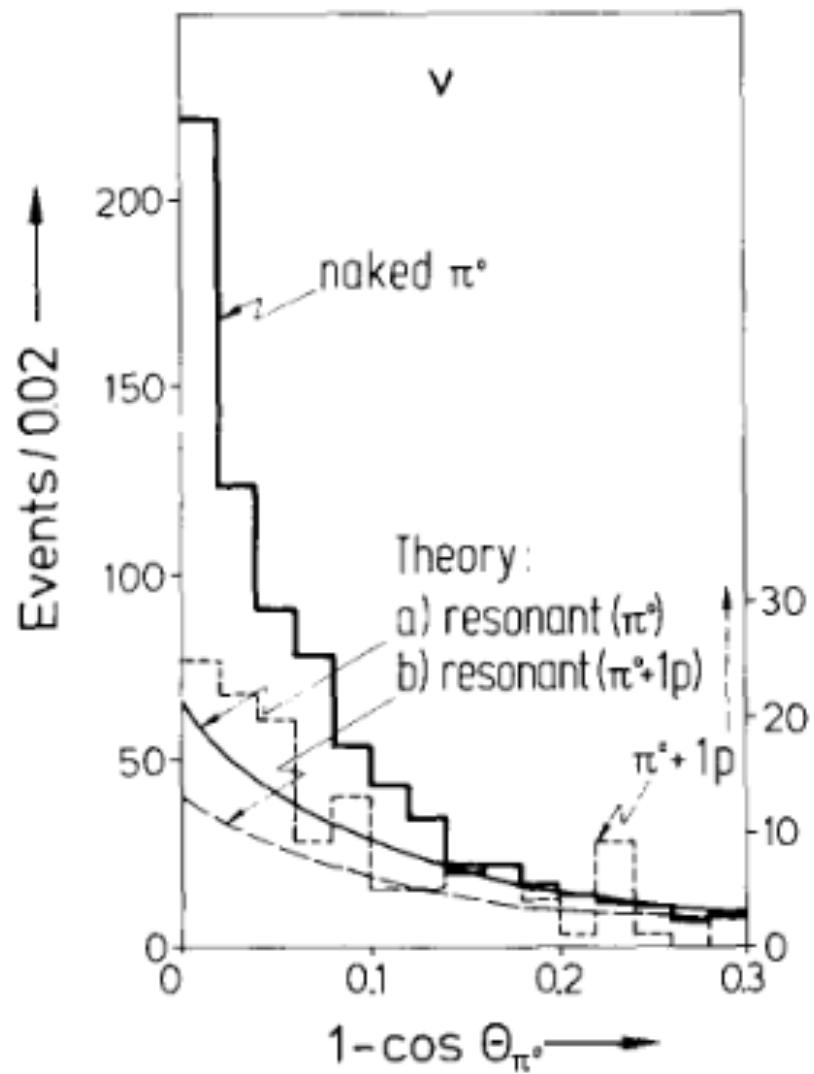
Pion production at small Q^2 .

Coherent pion Production.

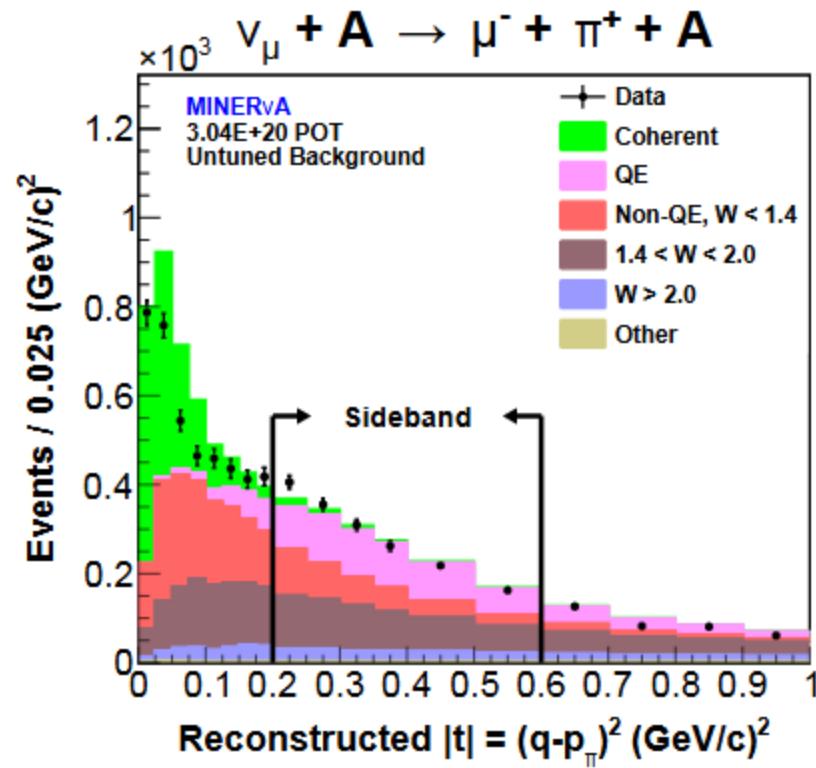
Searches of new neutrinos in the SBLv with light masses
 $m < \text{Mev/ } c^2$.

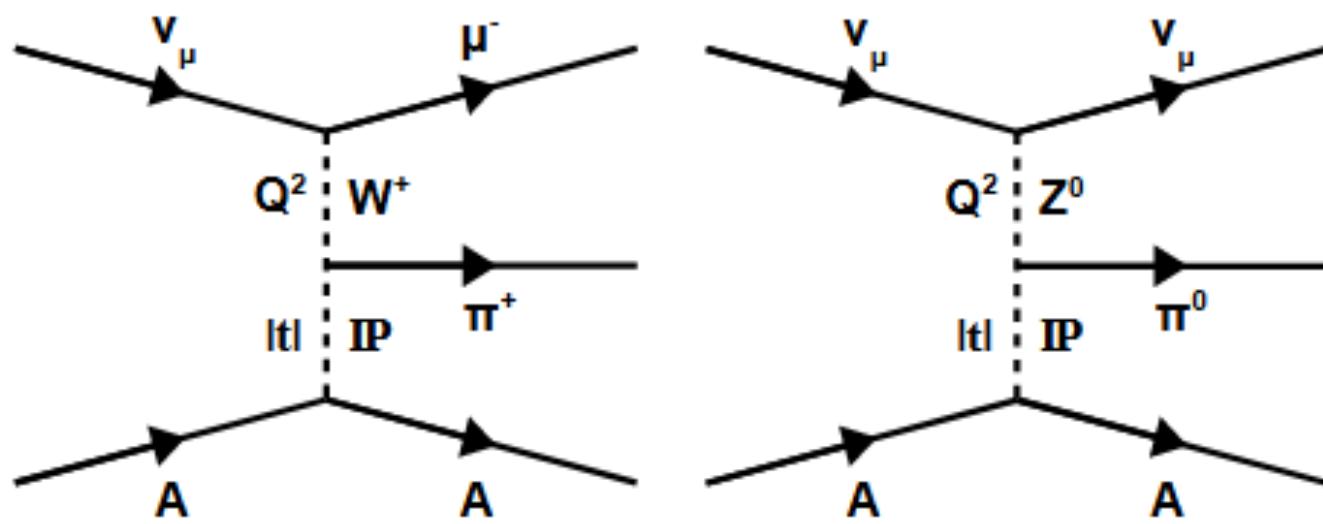
Coherent pion production





From Minerva





- Description and conditions for coherent Pion Production in Neutrino Scattering ($C\pi\nu NS$)
 - i. Polarization Vectors.
 - ii. PCAC.
 - iii. Coherence.
 - iv. A-dependence.
 - v. Using Carbon data instead of Nuclear model.

- The lepton current

$$J_\rho = \bar{u}(k_2)\gamma^\rho(1 - \gamma_5)u(k_1)$$

has three helicity $|1, \lambda\rangle$ polarizations with $q_\mu J^\mu = 0$,
and a scalar component proportional to q_μ .

For $C\pi\nu NS$ the dominant polarization is $\epsilon_\mu(\lambda=0)$.
I follow this in order to include the muon mass at low energies.

$$J_\mu(k', k) = C_0 \varepsilon_\mu^{long} + \sum C_\lambda \varepsilon_\mu(\lambda)$$

$$\varepsilon_\mu^{long} = \frac{1}{\sqrt{Q^2}}(q_0, 0, 0, q_3)$$

$$\varepsilon_\mu(\lambda) = \frac{1}{\sqrt{Q^2}}(q_3, 0, 0, q_0), \quad \text{for } \lambda = 0$$

$$= \frac{1}{\sqrt{2}}(0, 1, +i, 0), \quad \text{for } \lambda = +1$$

$$= \frac{1}{\sqrt{2}}(0, 1, -i, 0), \quad \text{for } \lambda = -1$$

$$\varepsilon_\mu(\lambda) q^\mu = 0$$

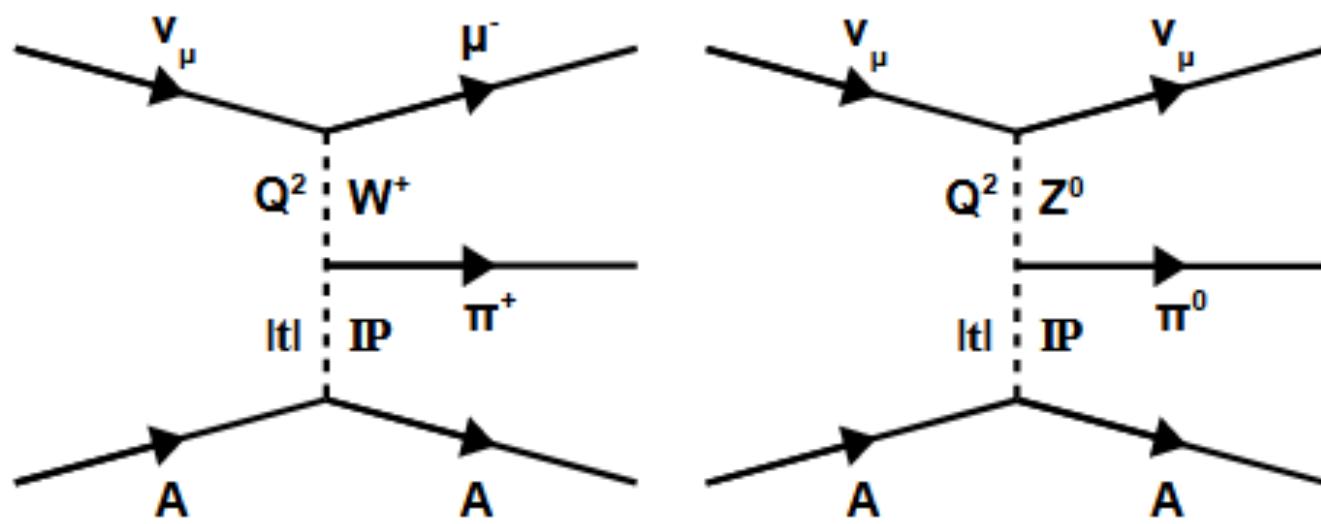
Coefficients of polarization tensor

$$L_{\mu\nu} = J_\mu \times J_\nu$$

$$\begin{aligned} K_\mu K'_\nu + K'_\mu K_\nu - g_{\mu\nu} K^* K' + i \epsilon_{\mu\nu\alpha\beta} K_\alpha K'_\beta \\ = L_{00} \epsilon_\mu(0) \epsilon_\nu(0) + \dots \end{aligned}$$

$$\tilde{L}_{00} = 4 \left[\frac{[Q^2(2E_\nu - \nu) - \nu m_\mu^2]^2}{Q^2(Q^2 + \nu^2)} - Q^2 - m_\mu^2 \right]$$

$$\tilde{L}_{00} \rightarrow \frac{2Q^2}{Q^2 + \nu^2} \left[4E_\nu E' - (Q^2 + m_\mu^2) - \frac{m_\mu^2}{Q^2} \nu^2 \right]$$



Coherent pion production

PART I: Gounaris, Kartavtsev, Paschos [**PRD74** (2006) 054007]

$$-iA_\rho^+ = \frac{f_\pi \sqrt{2} q_\rho}{Q^2 + m_\pi^2} T(\pi^+ N \rightarrow \pi^+ N) - R_\rho$$

The first amplitude is the pion pole, which varies rapidly for small Q^2 and R^ρ is the amplitude for the rest which is a smooth function of Q^2 .

PCAC gives the relation

$$-iq^\rho A_\rho^+ = \frac{\sqrt{2} f_\pi m_\pi^2}{Q^2 + m_\pi^2} T(\pi^+ N \rightarrow \pi^+ N)$$

while the definition of the amplitudes gives

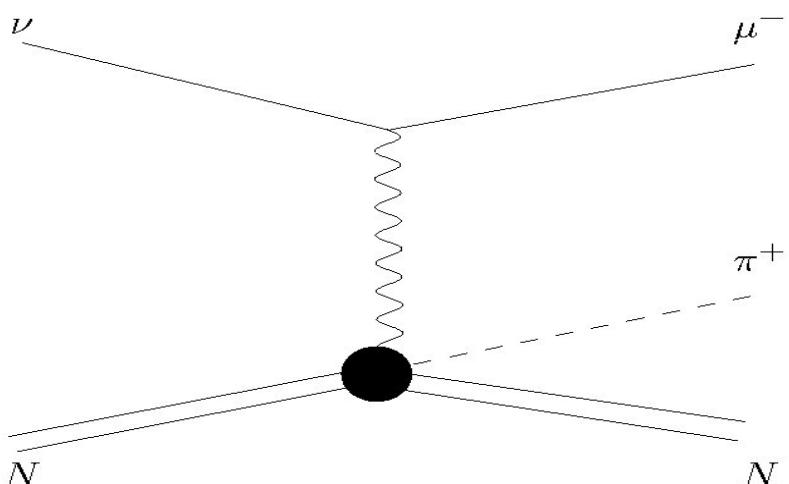
$$-iq^\rho A_\rho^+ = -\frac{\sqrt{2} f_\pi Q^2}{Q^2 + m_\pi^2} T(\pi^+ N \rightarrow \pi^+ N) - q^\rho R_\rho$$

Comparing the last two equations $\Rightarrow (A) = (B)$

$$q^\rho R_\rho = -\sqrt{2} f_\pi T(\pi^+ N \rightarrow \pi^+ N)$$

Factor Loo for Paschos+ Schalla vs Berger Sehgal

E=1 GeV	Q^2 =0.010GeV^2			Q^2 =0.10 GeV^2	
v GeV	Q^2 /min	Pasc+sch	BS	Pasc+Sch	BS
0.20	0.0028	.329	.215	.150	.189
0.25	0.0037	.240	.161	.140	.163
0.30	0.0048	.171	.122	.130	.136
0.35	0.0061	.119	.095	.110	.117
0.40	0.0074	.079	.073	.095	.097
0.45	0.0091	.047	.056	.085	.078
0.50	0.0112	.022		.070	.064
0.55	0.136			.058	.051



$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left\{ \frac{\sqrt{2}f_\pi}{\sqrt{Q^2}} T(\pi^+ N \rightarrow \pi^+ N) + c_R \varepsilon_\mu^R \cdot \langle \pi N | V^\mu - A^\mu | N \rangle + c_L \varepsilon_\mu^L \cdot \langle \pi N | V^\mu - A^\mu | N \rangle \right\}$$

- The pion pole term disappears exactly because $\varepsilon_\mu^\lambda q^\mu = 0$
- For $Q^2 \approx (\text{few}) \cdot m_\pi^2$ the first term dominates

FOR NEUTRAL CURRENTS

$$\begin{aligned} \frac{d\sigma}{dQ^2 d\nu dt} &= \frac{G_F^2 \nu}{4(2\pi)^2 E_1^2} \left[\frac{f_\pi^2}{Q^2} \tilde{L}_{00} \frac{d\sigma}{dt}(\pi^+ N \rightarrow \pi^+ N) \right. \\ &\quad \left. + \frac{\tilde{L}_{RR} + \tilde{L}_{LL}}{2} \left\{ \frac{(1 - 2s_W^2)^2}{2\pi\alpha} \frac{d\sigma}{dt}(\gamma N \rightarrow N\pi^0) + \frac{d\sigma}{dt}(A_T^+ N \rightarrow \pi^+ N) \right\} \right] \end{aligned}$$

where $\tilde{L}_{00}, \tilde{L}_{RR}, \tilde{L}_{LL}$ are density matrix elements given in GKP [PRD74 (2006) 054007]

$$\tilde{u},\tilde{v}=(E+E'\pm|\pmb{q}|)/2E.$$

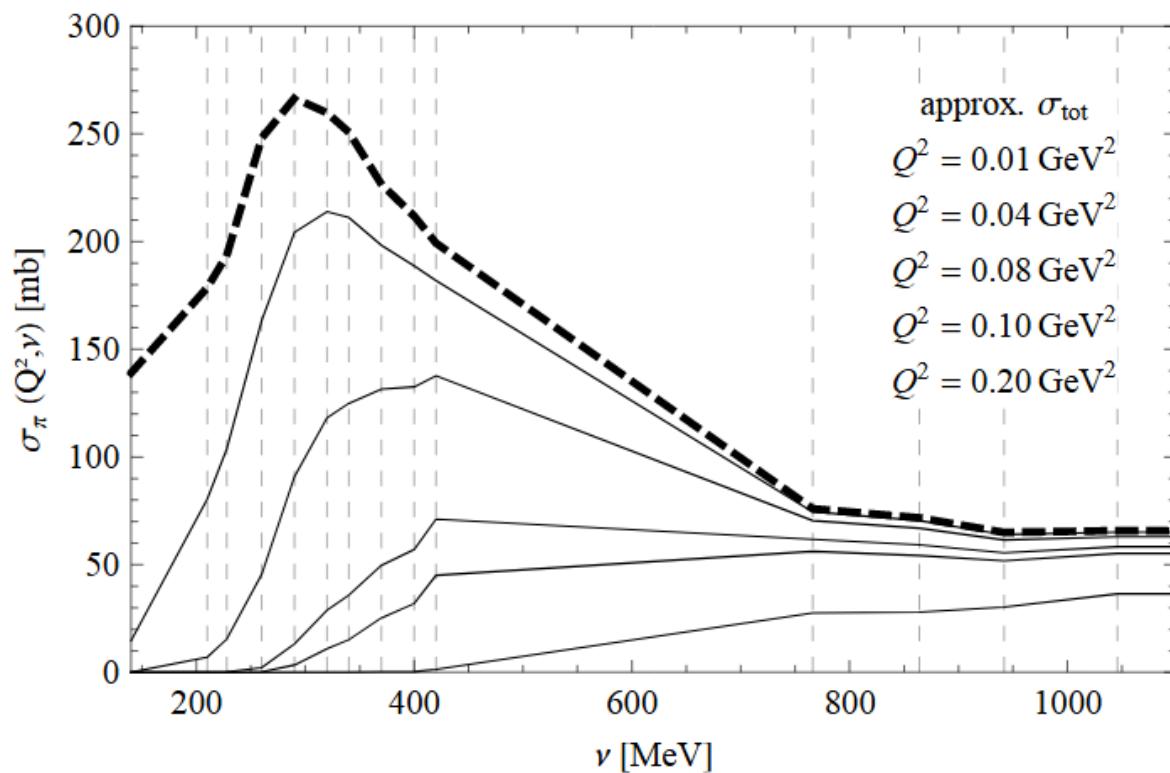
$$\frac{d\sigma^{CC}}{dQ^2dy}=\frac{G_F^2\cos^2\theta_C}{4\pi^2}\kappa E\frac{Q^2}{|q|^2}\left[u^2\sigma_L+v^2\sigma_R+2uv\sigma_S\right]$$

$$\left(\frac{Q^2 + m_\pi^2}{2\nu} \right)^2 \leq |t| \leq \infty$$

or better $|t| < 1/R^2$

$$\frac{d\sigma_\pi}{dt} = a \exp[-b|t|]$$

$$\sigma_\pi(Q^2, \nu) = \int_{t_{min}}^{\infty} \frac{d\sigma_\pi}{dt} dt$$



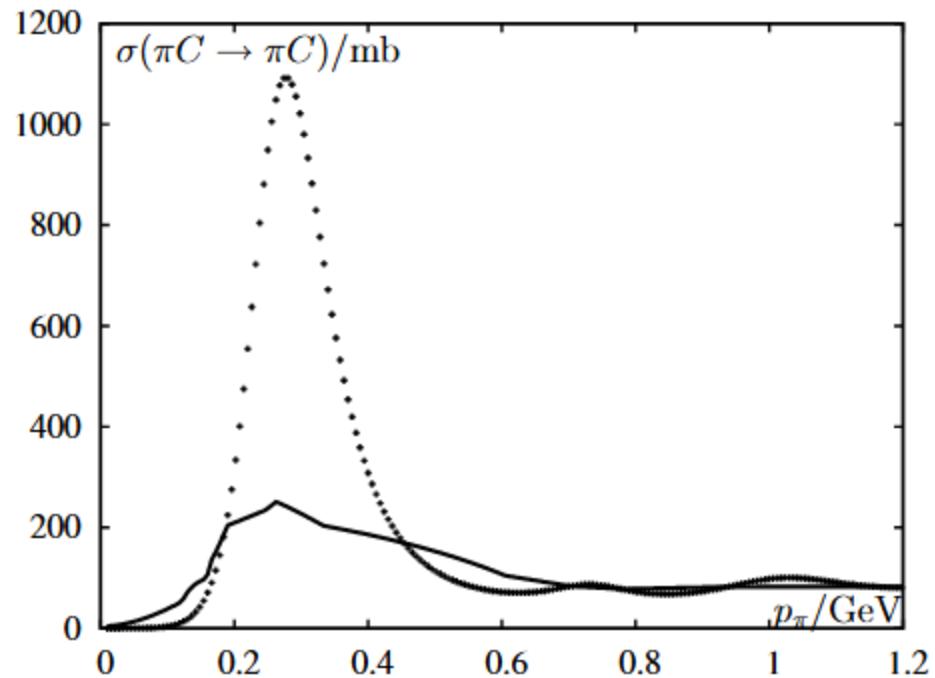


FIG. 2. The Rein-Sehgal (dashed line) and Berger-Sehgal

Summary of Conditions

- PCAC requires $Q^2 = 0 (m^2_\pi)$; a few m^2_π .
this is satisfied by introducing a form factor $F(q^2) = M_A^2/(Q^2 + M_A^2)$
- Low $\epsilon_\mu(0)\epsilon_\nu(0)$ dominance $v^2 \gg Q^2$
- Coherence $|t| < 1/R^2$ and large exponential decrease
- Checking the A dependence

REACTIONS POWER LAWS

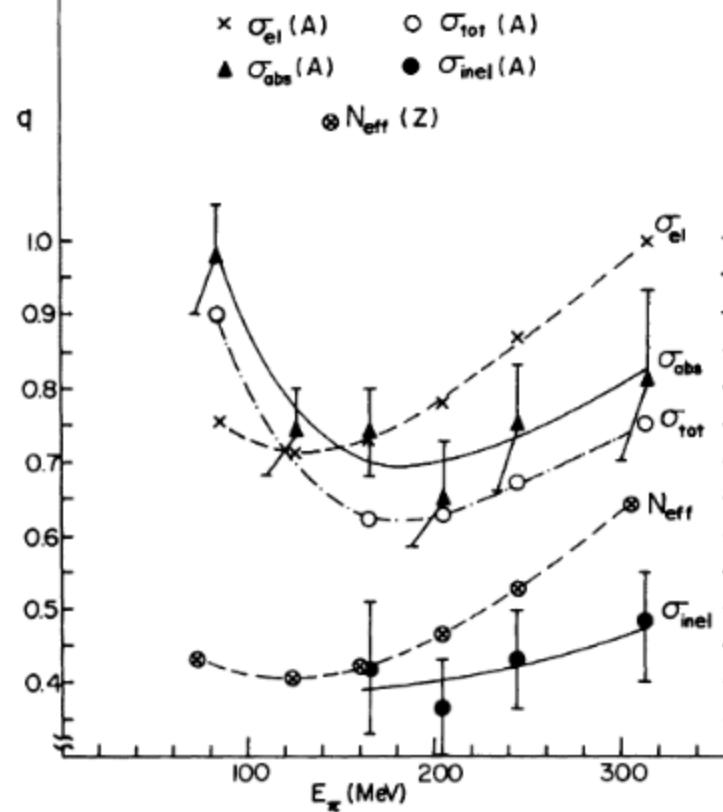
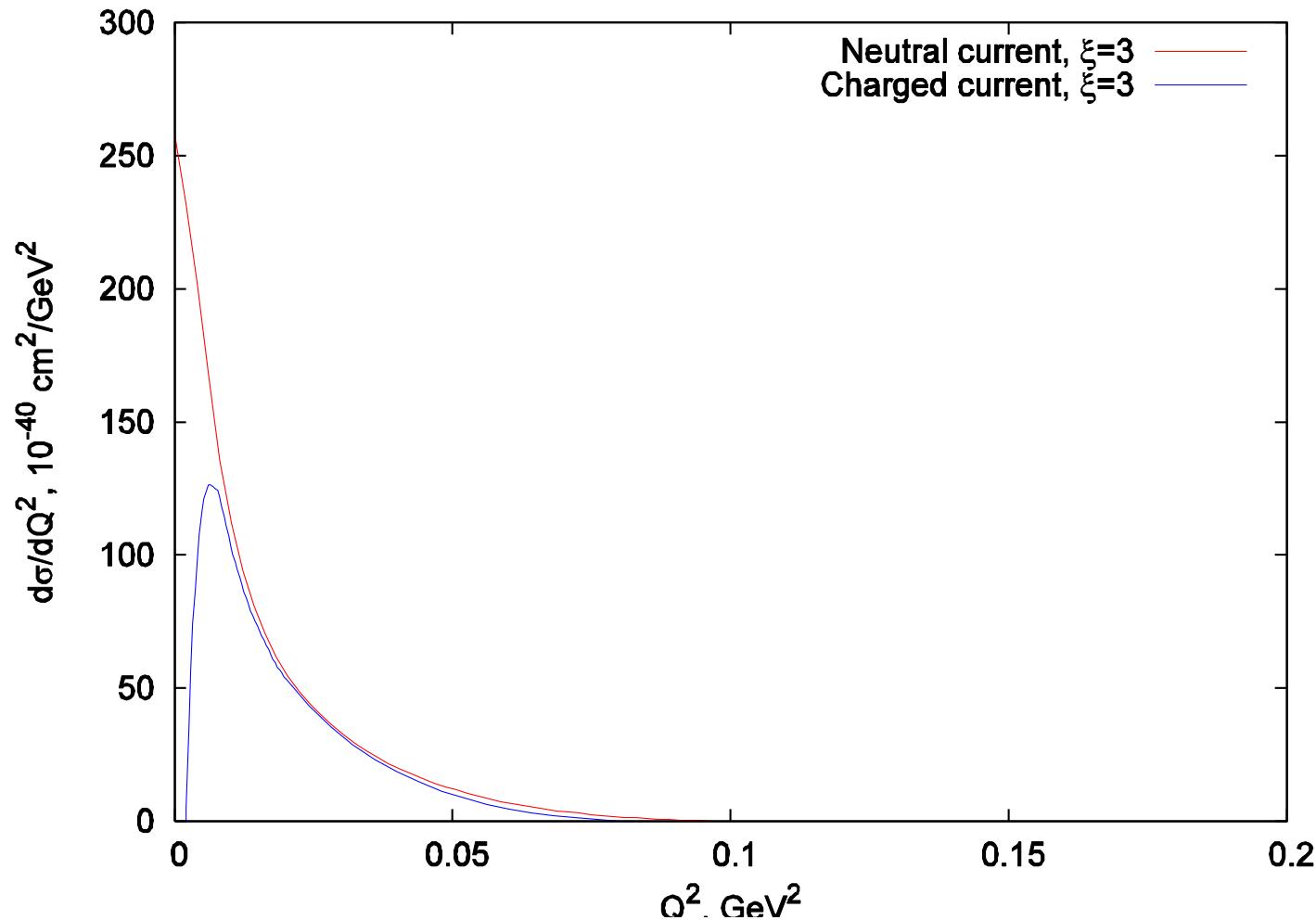
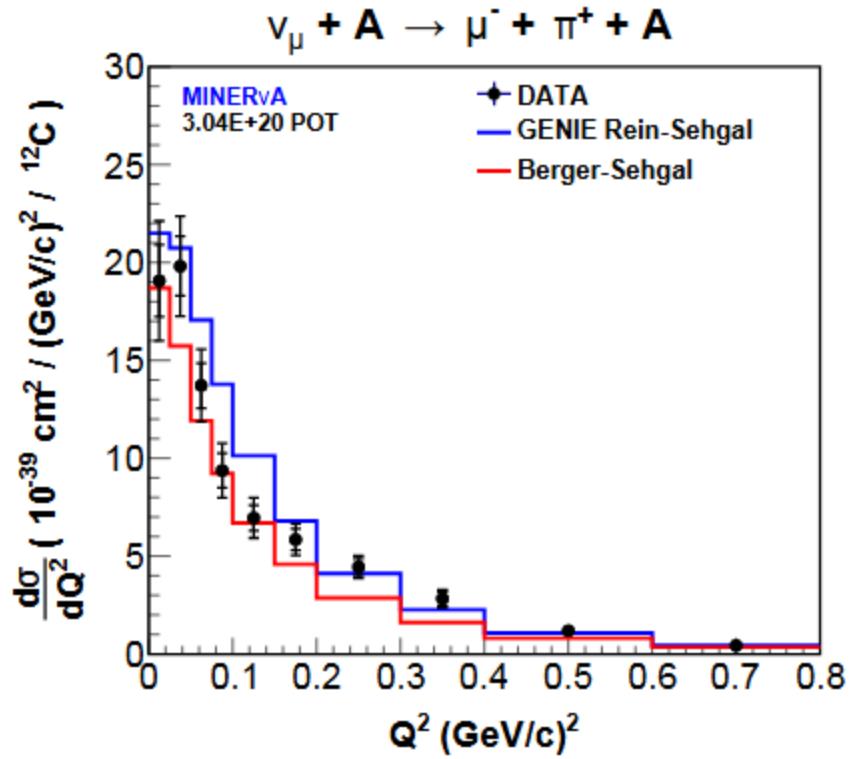
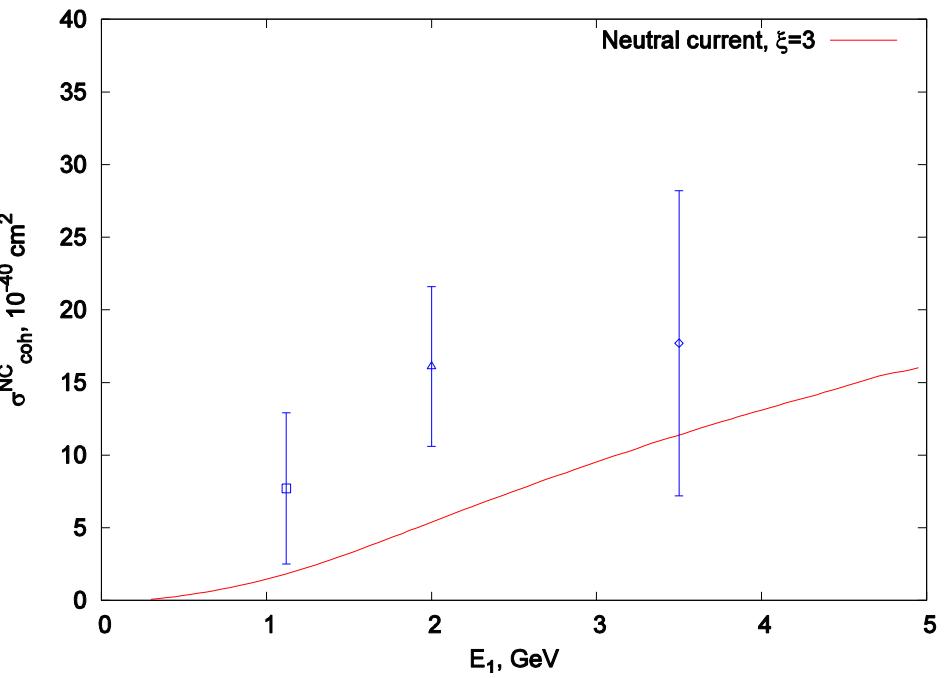
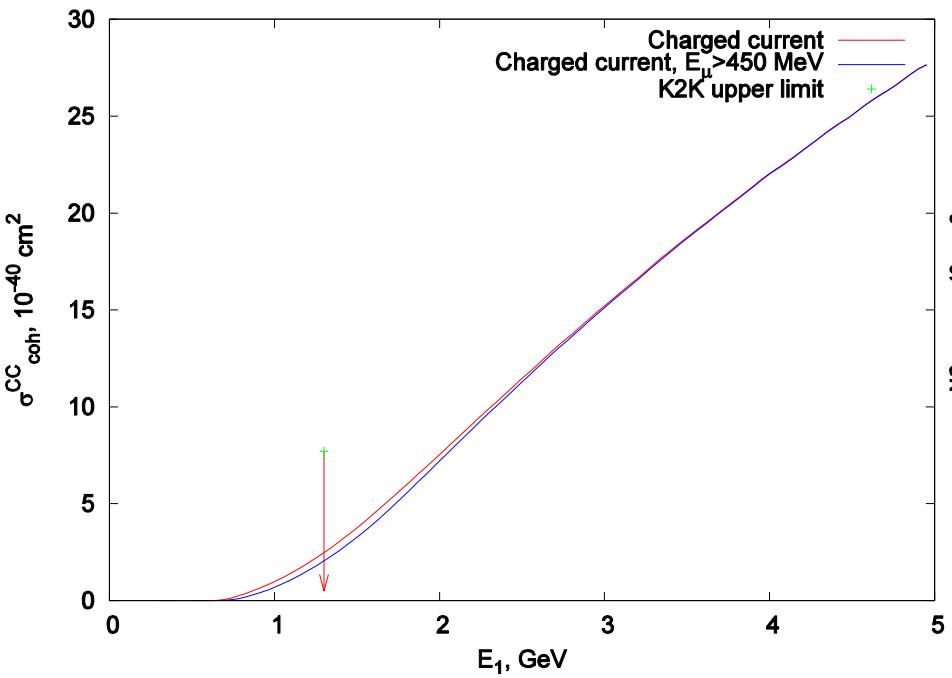


FIG. 10. Exponents of least squares fits to power laws for the cross sections as a function of A and of

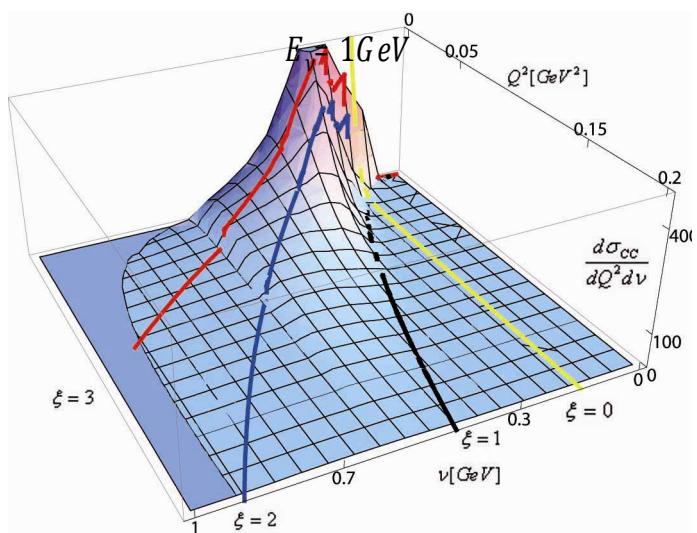
Coherent for CC and NC reactions



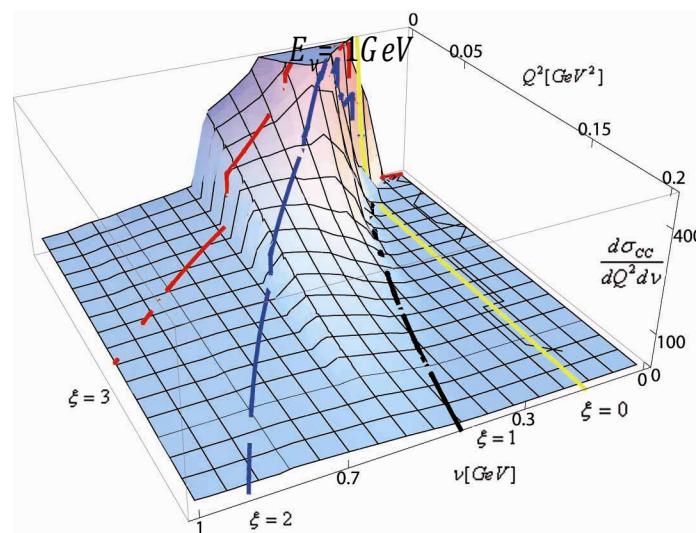




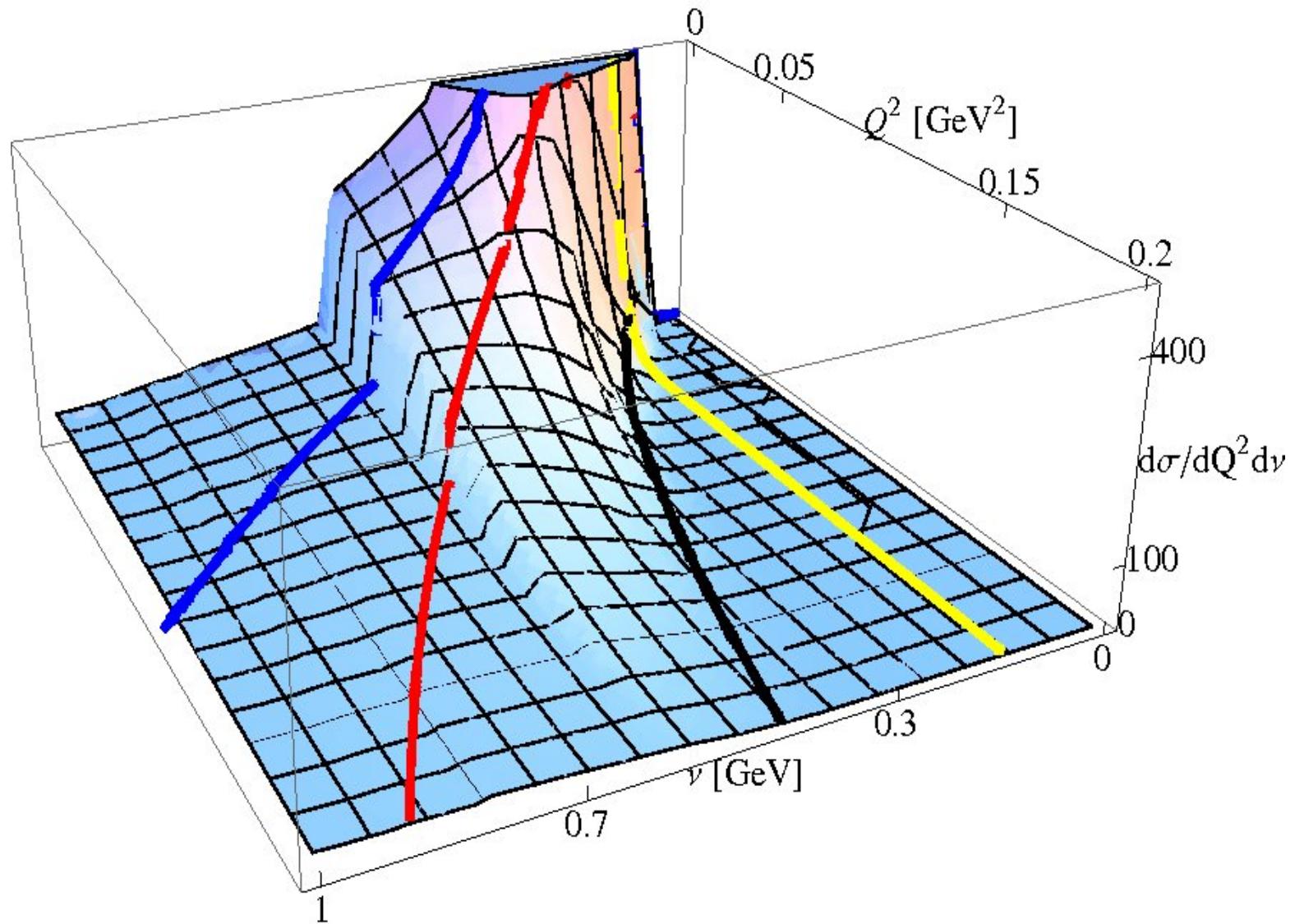
Charged current differential cross section



$$E_\nu = 1 \text{ GeV}$$



$$E_\nu = 10 \text{ GeV}$$



Cross sections

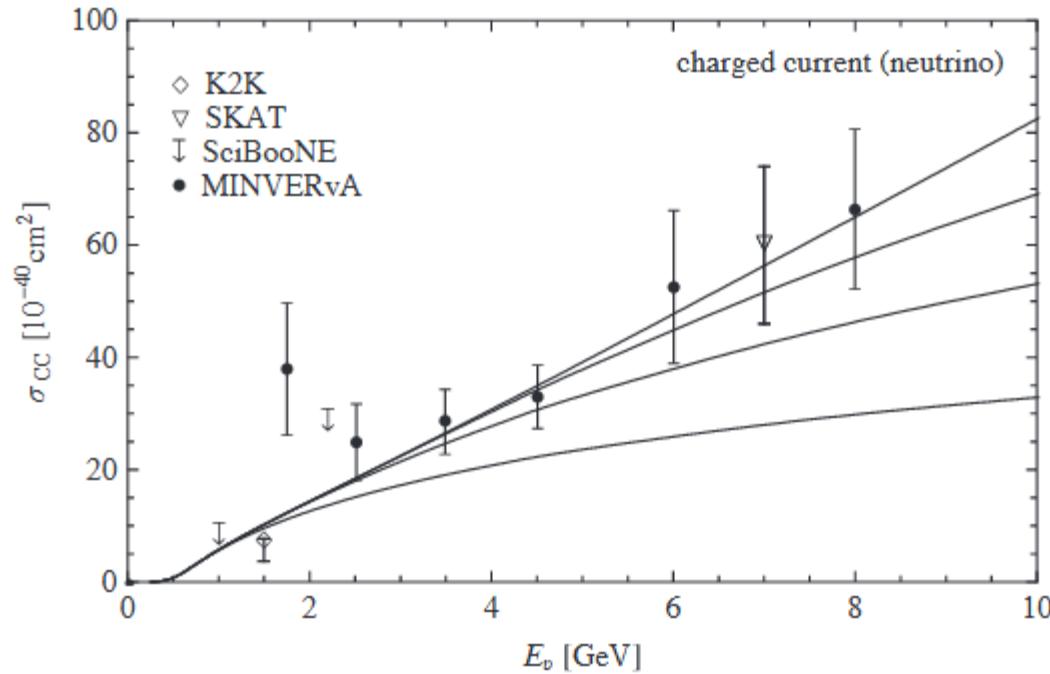


figure 2: Integrated charged current cross section with $Q^2_{\text{max}} = 0.2, 0.5, 1.0$ and 4.4 GeV^2 (bottom to top). The curves are from [4] and the data from [16,17,18,19].

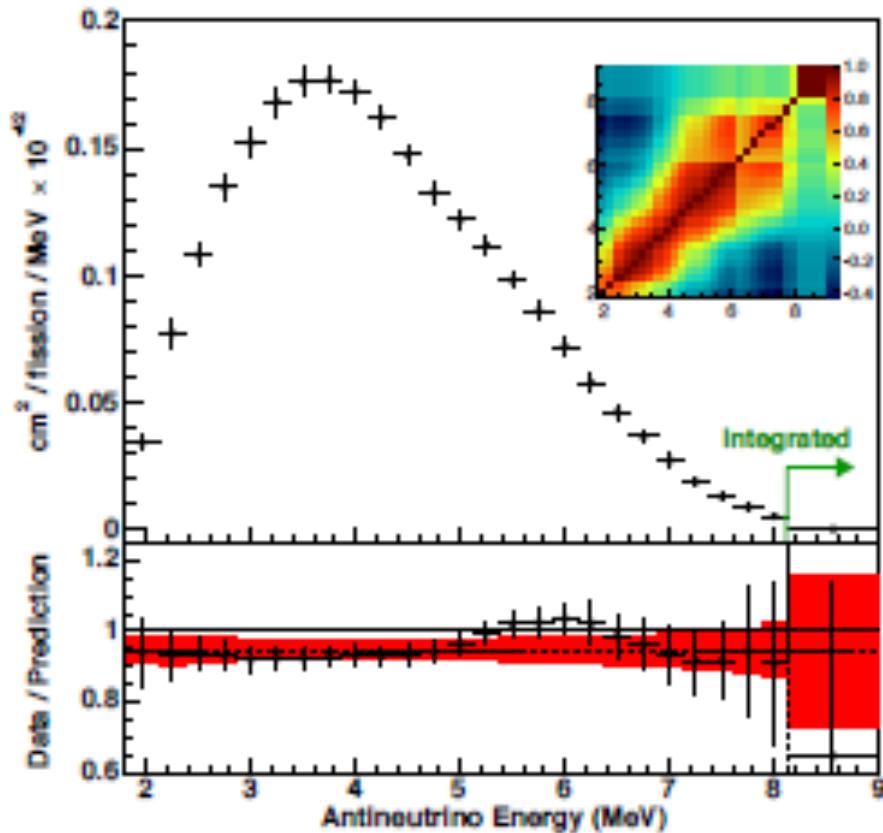
Development of the state $|\nu_e(t)\rangle$

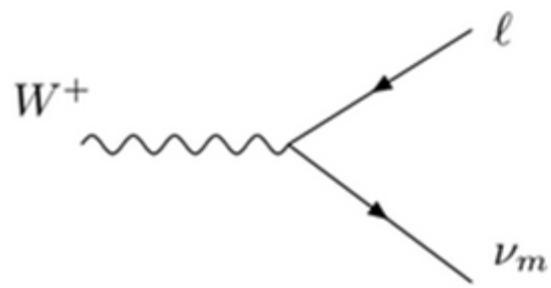
$$|\nu_e(t)\rangle = \sum_l U_{el} \phi_l(t) = U_{e1} \phi_1(t) + \dots = \\ U_{e1} \{ U_{1e} |\nu_e\rangle + U_{1\mu} |\nu_\mu\rangle + \dots + V_{1N} |N_R\rangle \} \exp(-iE_1 t) + \dots \exp(-iE_2 t) + \dots$$

In the time development of each flavor state, the lowest mass eigenstate brings in components of the new-flavor states. From the reactor anomaly we estimate

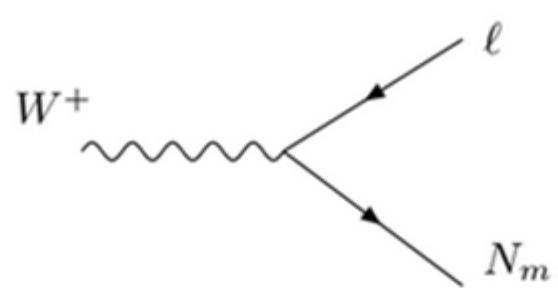
$$V_{1N} \sim 0.10 \text{ is possible}$$

Several anomalies suggest new sterile neutrinos

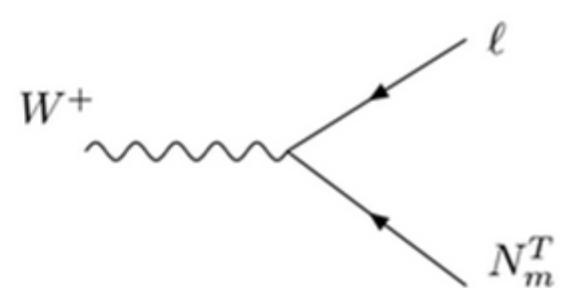




$$-i \frac{g}{\sqrt{2}} U_{\ell m}^* \gamma^\mu P_L$$



$$-i \frac{g}{\sqrt{2}} V_{\ell m}^* \gamma^\mu P_L$$



$$-i \frac{g}{\sqrt{2}} V_{\ell m}^* C \gamma^\mu P_L$$

New interactions

- $N\mu + e \rightarrow \mu^- + \nu$ (antineutrino beam)



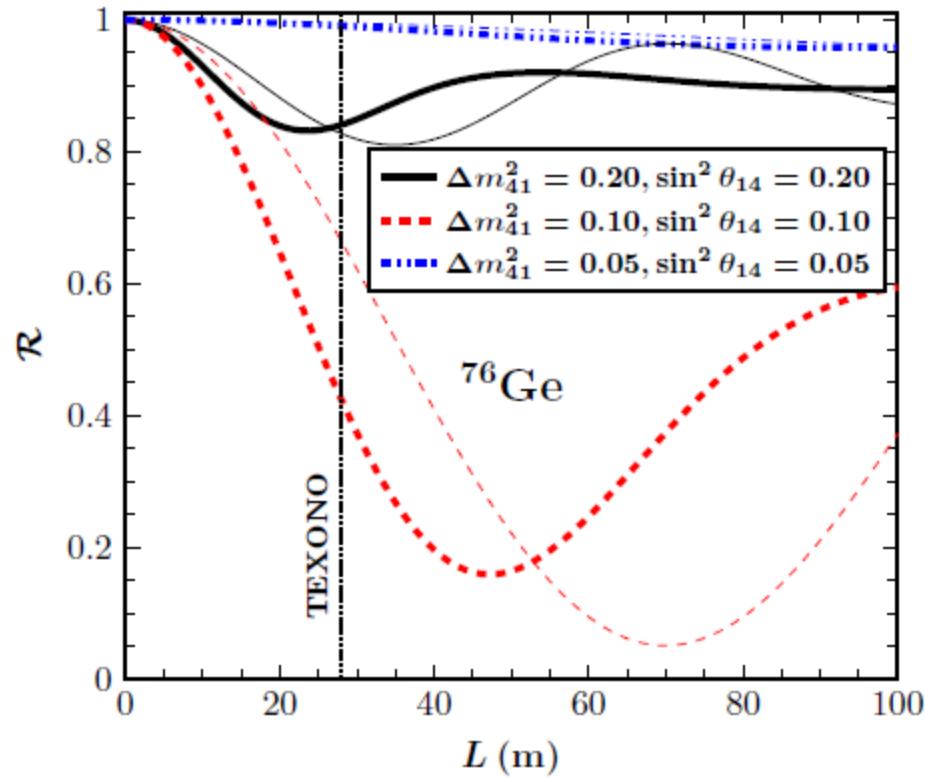
Down stream the muon neutrinos develop small components of the other and the new states.

$$P_{ee} \simeq 1 - \cos^4 \theta_{14} \sin^2 2\theta_{13} \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E_\nu} \right) \\ - \sin^2 2\theta_{14} \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E_\nu} \right).$$

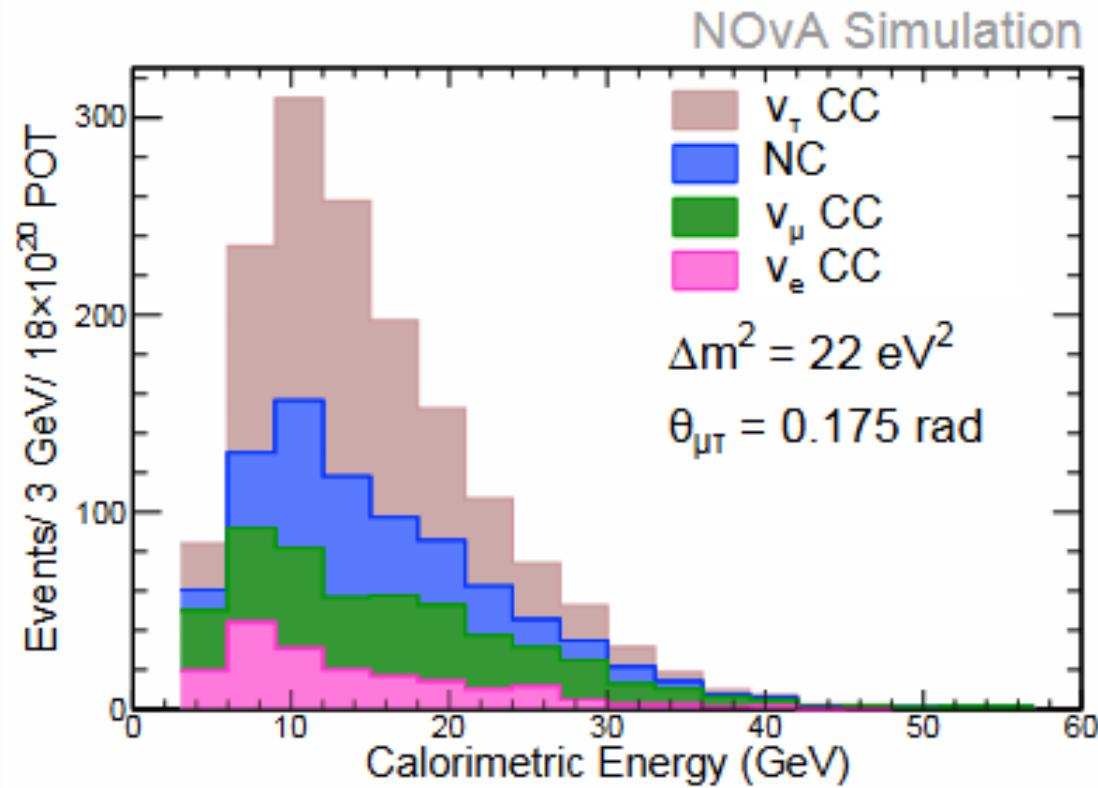
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$$\text{PeN} = \sin^2 2\theta_{14} \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E_\nu} \right)$$

Kosmas et al. arXiv:1710.00295



arXiv:1710.00295; Keloth et al. (L.
Suter and R. Plunkett ---Fermilab)



Summary

- a. Coherent pion production is a rare but interesting reaction that may help other investigations.
- b. There are still open issues to understand: the modelling of the Q^2 - and the A-dependence.
- c. $d\sigma / (dQ^2 dv)$ are more informative
- d. The time development of the known neutrinos have components of heavy flavors (sterile and Majorana, ...) which are investigated in recent articles, especially for the promising neutrino program at Fermilab.

References for theory

1. Rein and Sehgal , Nucl.Phys. B223, 1983
2. Gounaris, Kartavtsev and EAP, Phys.Rev. D74
2006
3. Berger and Sehgal , Phys.Rev.D79, 2009
4. EAP and Schalla, Phys. Rev. D80, 2009
5. B. Z. Kopeliovich : Neutrino production of pions
off nuclei,"<http://www.fis.utfsm.cl/np>

END

- Thank you !

Anatomy of cross sections

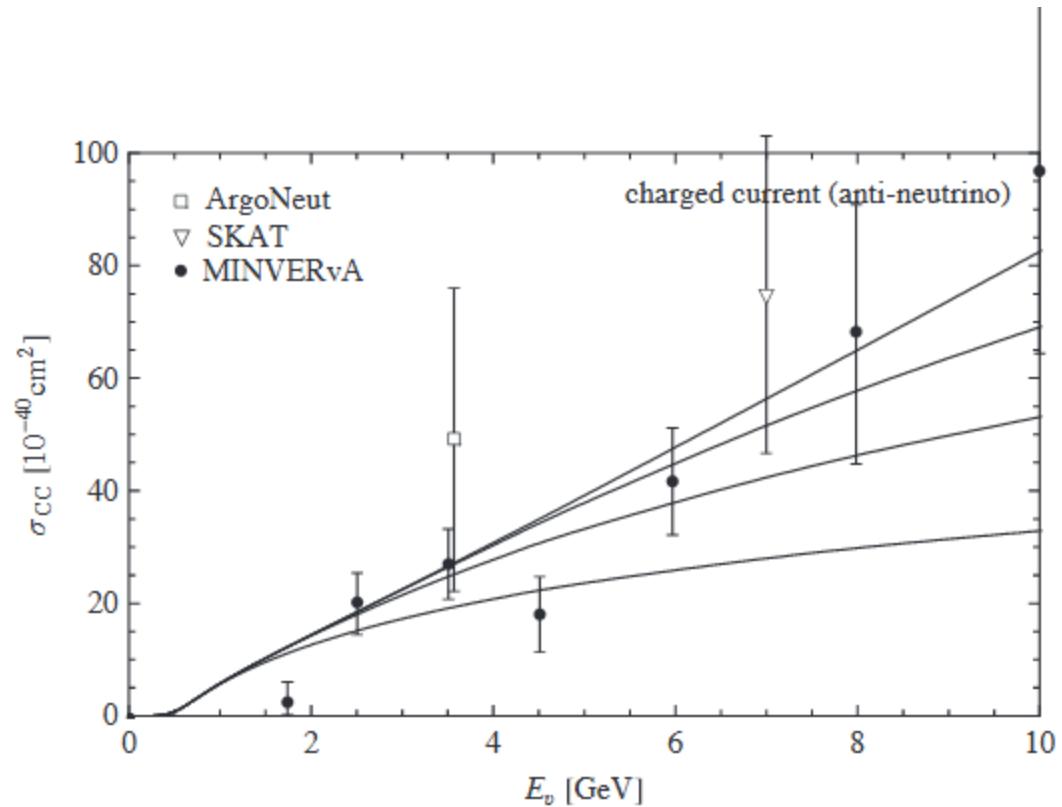
- $\sigma_{\text{dis}} = 0.65 \times 10^{-38} E_v (\text{Gev}) \text{ cm}^2$

for $E_v = 10 \text{ GeV}$: $6.5 \times 10^{-38} \text{ cm}^2$

- $\sigma_{\text{qe}} = 0.50 \times 10^{-38} \text{ cm}^2$

- $\sigma(\Delta) = 0.60 \times 10^{-38} \text{ cm}^2$

- $\sigma(\text{coh}, \pi) = (1.0 \text{ to } 3.0) \times 10^{-40} \text{ cm}^2 / \text{Nucleus}$



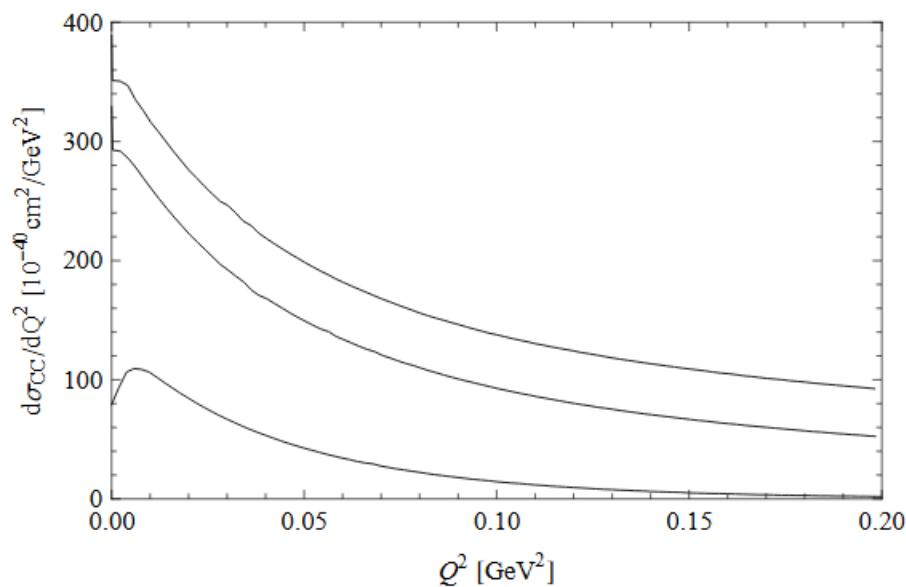
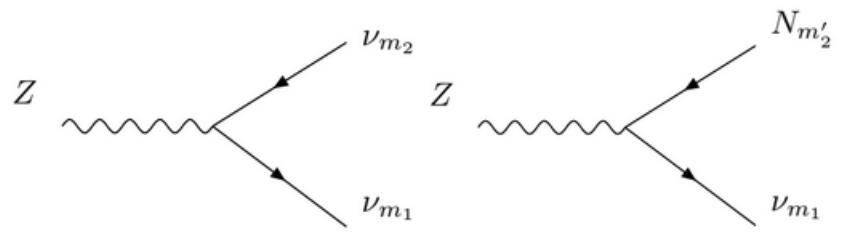
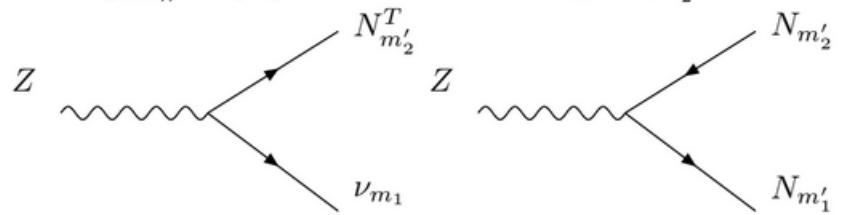


Fig. 8: Differential charged current cross section for $\xi = 0$ at $E_\nu = 1, 5$ and 10 GeV .



$$-i \frac{g}{2 \cos W} U_{m_1 m_2}^{\nu \nu} \gamma^\mu P_L$$



$$-i \frac{g}{2 \cos W} U_{m_1 m'_2}^{\nu N} \gamma^\mu P_L C$$



$$-i \frac{g}{2 \cos W} V_{m'_1 m'_2}^{NN} \gamma^\mu P_L$$

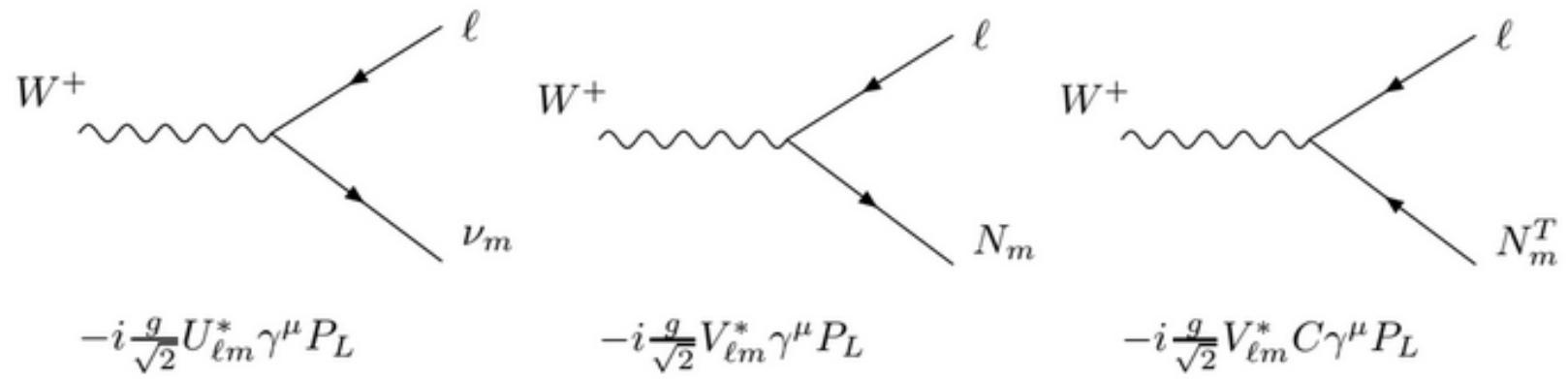


Figure 25: Feynman rules for the charged current vertices in terms of the neutrino mass eigenstates, as given in Eq. (A.22).

$$\tilde{L}_{00}=4\left[\frac{[Q^2(2E_{\nu}-\nu)-\nu m_{\mu}^2]^2}{Q^2(Q^2+\nu^2)}-Q^2-m_{\mu}^2\right]$$

$$\cdots \cdots \cdots$$

$$W_2^A(Q^2,\nu)=\frac{2f_\pi^2}{\pi}\frac{\nu}{Q^2+\nu^2}\sigma(\pi^+p\rightarrow X^{++})$$

$$\epsilon_s^\mu = \frac{q^\mu}{\sqrt{Q^2}} \qquad \epsilon_\mu(\lambda=0) = \frac{1}{\sqrt{Q^2}} \left(|\vec{q}|,\; 0,\; 0,\; q_0 \right)$$

$$\langle \Delta^{++} \, | \, {\cal A}_\mu^+ \, | \, p \rangle = \sqrt{3} \, \overline{\psi}_\lambda(p') i g_\mu^\lambda C_5^A(Q^2) u(p) + \sqrt{3} \frac{i f_\pi q_\mu}{q^2 - m_\pi^2} \, \langle \Delta \, | \, j_\pi \, | \, p \rangle$$

$$\begin{aligned} iq^\mu \, \langle \Delta^{++} \, | \, {\cal A}_\mu^+ \, | \, p \rangle &= \sqrt{3} \, \overline{\psi}_\lambda(p') q^\lambda C_5^A(Q^2) u(p) + \sqrt{3} \frac{f_\pi q^2}{q^2 - m_\pi^2} \, \langle \Delta \, | \, j_\pi \, | \, p \rangle \\ &= \sqrt{3} \frac{f_\pi m_\pi^2}{q^2 - m_\pi^2} \, \langle \Delta \, | \, j_\pi \, | \, p \rangle \, . \end{aligned}$$

$$C_5^A(Q^2) \overline{\psi}_\lambda(p') q^\lambda u(p) = - f_\pi \, \langle \Delta \, | \, j_\pi \, | \, p \rangle$$

$$\left\langle X^{++}\right.\big|\mathcal{A}^+_\mu\left.\vphantom{X^{++}}\right|p\big\rangle=\mathcal{R}_\mu+\sqrt{3}\frac{if_\pi q_\mu}{q^2-m_\pi^2}\left\langle X^{++}\right.\big|j_\pi\left.\vphantom{X^{++}}\right|p\big\rangle$$

$$q^\mu \mathcal{R}_\mu = -if_\pi \left\langle X^{++}\right.\big|j_\pi\left.\vphantom{X^{++}}\right|p\big\rangle$$

$${\cal A}(W^+n\rightarrow X^+)={\cal A}(W^-p\rightarrow X^0)$$

$$\begin{aligned}\epsilon^\mu(\lambda=0)\left\langle\Delta^{++}\right.\big|\mathcal{A}_\mu\left.\vphantom{\Delta^{++}}\right|p\big\rangle&=\epsilon^\mu(\lambda=0)i\sqrt{3}\overline{\psi}_\mu(p')C_5^A(Q^2)u(p)\\&\approx i\sqrt{3}\frac{q^\mu}{\sqrt{Q^2}}\overline{\psi}_\mu(p')C_5^A(Q^2)u(p)+\mathcal{O}\left(\frac{Q^2}{\nu^2}\right)\\&\approx-\frac{f_\pi\sqrt{2}}{\sqrt{Q^2}}{\cal A}(\pi^+p\rightarrow\Delta^{++})\end{aligned}$$

$$\frac{{\rm d}\sigma^A}{{\rm d}Q^2{\rm d}\nu}=\frac{G_F^2|V_{ud}|^2}{2\pi}\frac{1}{4\pi}\frac{\nu}{E_\nu^2}\frac{f_\pi^2}{Q^2}\left\{\tilde L_{00}+2\tilde L_{l0}\frac{m_\pi^2}{Q^2+m_\pi^2}+\tilde L_{ll}\left(\frac{m_\pi^2}{Q^2+m_\pi^2}\right)^2\right\}\sigma(\pi^+p\rightarrow X^{++})$$

$$\tilde{L}_{00}=4\left[\frac{[Q^2(2E_{\nu}-\nu)-\nu m_{\mu}^2]^2}{Q^2(Q^2+\nu^2)}-Q^2-m_{\mu}^2\right]$$

$$\tilde{L}_{00}\rightarrow \frac{2Q^2}{Q^2+\nu^2}\left[4E_{\nu}E'-(Q^2+m_{\mu}^2)-\frac{m_{\mu}^2}{Q^2}\nu^2\right]$$

$$W_2^A(Q^2,\nu)=\frac{2f_\pi^2}{\pi}\frac{\nu}{Q^2+\nu^2}\sigma(\pi^+p\rightarrow X^{++})$$

$$\left[g_A(Q^2)\right]^2 + \int_{\nu_{th}}^\infty {\rm d}\nu \left[W_{2,\nu n}^A(Q^2,\nu) - W_{2,\nu p}^A(Q^2,\nu)\right] = 1$$

$$\left[g_A(Q^2)\right]^2 + \frac{2f_\pi^2}{\pi}\int_{\nu_{th}}^\infty {\rm d}\nu\frac{\nu}{Q^2+\nu^2}\left[\sigma^{\pi^+n}(\nu)-\sigma^{\pi^+p}(\nu)\right]=1$$

$$\left\langle \Delta^{++} \right. | \, {\cal A}_\mu^+ \left. | \, p \right\rangle = \sqrt{3} \, \overline{\psi}_\lambda(p') i g_\mu^\lambda C_5^A(Q^2) u(p) + \sqrt{3} \frac{i f_\pi q_\mu}{q^2 - m_\pi^2} \left\langle \Delta \right. | \, j_\pi \left. | \, p \right\rangle$$

Cross section at small Q^2

- Cross Section with zero helicity and longitudinal polarization. The formula is exact without transverse polarizations.

$$\frac{d\sigma^A}{dQ^2 d\nu} = \frac{G_F^2 |V_{ud}|^2}{2\pi} \frac{1}{4\pi} \frac{\nu}{E_\nu^2} \frac{f_\pi^2}{Q^2} \left\{ \tilde{L}_{00} + 2\tilde{L}_{l0} \frac{m_\pi^2}{Q^2 + m_\pi^2} + \tilde{L}_{ll} \left(\frac{m_\pi^2}{Q^2 + m_\pi^2} \right)^2 \right\} \sigma(\pi^+ p \rightarrow X^{++}).$$

(10)

$$g_A(Q^2) = \frac{-1.26}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}$$

ν [GeV]	W [GeV]	$\sigma(\pi^+ p)$ [mb]	$\sigma(\pi^- p)$ [mb]	$\frac{\nu f_\pi^2 \tilde{L}_{00}}{E_\nu Q^2}$ [GeV]
0.20	1.118	16	12	0.329
0.25	1.159	77	30	0.240
0.30	1.199	189	67	0.171
0.35	1.238	175	63	0.119
0.40	1.275	95	37	0.079
0.45	1.311	60	28	0.047
0.50	1.347	42	26	0.022
0.55	1.381	31	28	0.002

ν [GeV]	W [GeV]	$\sigma(\pi^+ p)$ [mb]	$\sigma(\pi^- p)$ [mb]	$\frac{\nu f_\pi^2 \tilde{L}_{00}}{E_\nu Q^2}$ [GeV]
0.20	1.053	16	12	0.109
0.25	1.097	77	30	0.113
0.30	1.139	189	67	0.111
0.35	1.180	175	63	0.104
0.40	1.219	95	37	0.095
0.45	1.257	60	28	0.085
0.50	1.294	42	26	0.074
0.55	1.330	31	28	0.063
0.60	1.364	24	28	0.052
0.65	1.398	17	34	0.042